

Chiral NN interactions in nuclear matter

Boris Krippa

*Department of Physics and Astronomy, Free University of Amsterdam,
De Boelelaan 1081, 1081 HV Amsterdam.*

We consider an effective field theory of NN system in nuclear medium. The shallow bound states, which complicate the effective field theory analysis and lead to the large scattering length in the vacuum case do not exist in matter. We show that the next-to-leading order terms in the chiral expansion of the effective NN potential can be interpreted as corrections so that the expansion is systematic. It is pointed out however that it is still useful to treat the problem non-perturbatively since it may allow for the consideration of the nuclear systems with the density smaller than the normal nuclear matter one. The potential energy per particle is calculated. The possible directions in constructing the chiral theory of nuclear matter are outlined.

Effective Field Theory (EFT) has become a popular tool for studying nuclear interactions. EFT is based on the idea to use the Lagrangian with the appropriate effective degrees of freedom instead of the fundamental ones in the low-energy region (for review of EFT see, for example [1]). This Lagrangian should include all possible terms allowed by the symmetries of the underlying QCD. The states which can be treated as heavy, compared to the typical energy scale involved, are integrated out. They are hidden in the Low Energy Effective Constants (LEC's) of the corresponding Lagrangian. The physical amplitudes can be represented as the sum of certain graphs, each of them being of a given order in Q/Λ , where Q is a typical momentum scale and Λ is a parameter reflecting the scale of the short range physics. The relative contribution of each graph can roughly be estimated using chiral counting rules [2]. The relevant degrees of freedom in the nuclear domain are nucleons and pions. In the case of meson-meson [3] and meson-nucleon [4] interactions the perturbative chiral expansion can be organized in a consistent way. However, being applied to the NN system EFT encounters serious problem which is due to existence of the bound states near threshold [5]. It results in the large nucleon-nucleon scattering length and makes the perturbative expansion divergent. Weinberg suggested [5] to apply chiral counting rules to the certain class of the irreducible diagrams which should then be summed up to infinite order by solving the Lippmann-Schwinger (LS) equation. The irreducible diagrams can be treated as the effective potential in this case. Different aspects of the chiral NN problem have been discussed since then [6]. The concept of EFT has also intensively been used to

study nuclear matter [7–10]. In [7] the effective chiral Lagrangian was constructed and the “naturalness” of the effective coupling constants has been demonstrated. The possible counting rules for nuclear matter have been discussed in [9]. These two lines of development of the chiral nuclear physics are in some sense similar to the tendencies existed some time ago in conventional nuclear physics with the phenomenological two body forces. On the one hand, the phenomenological NN potentials were used to describe nucleon-nucleon cross sections and phase shifts. On the other hand, nuclear mean field approaches provided a reasonable description of the bulk properties of nuclear matter. The unification of these two approaches then led to the famous Bethe-Goldstone (BG) equation [11] for the G-matrix which is an analog of scattering T-matrix, satisfying the LS equation. It is therefore reasonable to follow the same strategy and, being equipped with the chiral theory of NN interaction in vacuum, try to construct the chiral G-matrix, describing the effective interactions of two nucleons in medium. One can easily see the qualitative difference between vacuum and medium cases. In nuclear medium because of Pauli blocking the intermediate states with the momenta less than Fermi momentum p_F are forbidden. Therefore, the nucleon propagator does not exhibit a pole. Moreover, the shallow bound or virtual NN states, which constitute the main difficulty of the problem in vacuum, simply do not exist in nuclear matter because of interaction of the NN pair with nuclear mean field. It means that the effective scattering length becomes considerably smaller compared to the vacuum one. The value of a scattering length is determined by the position of the singularity, nearest to the physical region. In the vacuum case, for example, the virtual deuteron bound state is very close to the NN threshold leading to the unnaturally large scattering length. The moderate value of the in-medium scattering length would indicate, in some sense, that the typical scale of the NN interactions gets “more natural” in nuclear matter.

We start from the standard nucleon-nucleon effective chiral Lagrangian which can be written as follows

$$\mathcal{L} = N^\dagger i \partial_t N - N^\dagger \frac{\nabla^2}{2M} N - \frac{1}{2} C_0 (N^\dagger N)^2 - \frac{1}{2} C_2 (N^\dagger \nabla^2 N) (N^\dagger N) + h.c. + \dots \quad (1)$$

We consider the simplest case of the NN scattering in the 1S_0 state and assume zero total 3-momentum of NN pair in the medium. The inclusion of the nonzero total 3-momentum does not really change anything qualitatively and only makes the calculations technically more involved. The G-matrix is given by

$$G(p', p) = V(p', p) + M \int \frac{dq q^2}{2\pi^2} V(p', q) \frac{\theta(q - p_F)}{M(\epsilon_1(p) + \epsilon_2(p')) - q^2} G(q, p), \quad (2)$$

Here ϵ_1 and ϵ_2 are the single-particle energies of the bound nucleons. They are affected by the nuclear mean field. In nuclear medium such corrections lead to the nucleon effective mass slightly different from that in free space. We used the value $M = 0.8M_0$, where M_0 is the nucleon mass in vacuum. One notes that this value is

close to one usually accepted in nuclear mean field theories. The standard strategy of treating the chiral NN problem in vacuum is the following. One computes amplitudes up to a given chiral order in the terms of the effective constants C_0 and C_2 which are then determined by comparing the calculated amplitude with some experimental data. Having these constants fixed one can calculate the other observables. We will follow the similar strategy in the nuclear matter case and proceed as follows. We choose exactly solvable separable potential with parameters adjusted to the value of the potential energy per particle in nuclear matter. Then we solve the BG equation with the effective constants C_0 and C_2 . The numerical values of these constants are determined comparing the phenomenological and EFT G-matrix at some fixed kinematical points. The check of consistency we used is the difference between C_0 's determined in the leading and subleading orders. If the difference between the values of C_0 needed to fit the data in leading and subleading order is of higher order then the procedure of truncation of the standard chiral expansion is justified. In the vacuum case the corresponding difference was found to be large [12]. Using a simple separable potential

$$V = -\lambda|\eta\rangle\langle\eta| \quad (3)$$

with the form factors

$$\eta(p) = \frac{1}{(p^2 + \beta^2)^{1/2}} \quad (4)$$

One can easily get

$$\frac{1}{T(k, k)} = V(k, k)^{-1} \left[1 - M_0 \int \frac{dq q^2}{4\pi^2} \frac{V(q, q)}{k^2 - q^2} \right] \quad (5)$$

The experimental values of scattering length a and effective radius r_e are

$$a = -23.71 \pm 0.013 \text{ fm} \quad r_e = 2.73 \pm 0.03 \text{ fm}. \quad (6)$$

These values can be reproduced if we choose

$$\lambda = 1.95 \quad \beta = 0.8 \text{ fm} \quad (7)$$

The solution of the BG equation for the separable potential is a simple generalization of the one for the LS equation

$$G(k, k) = -\eta^2(k) \left[\lambda^{-1} + \frac{M}{2\pi^2} \int dq q^2 \frac{\theta(q - p_F) \eta^2(q)}{k^2 - q^2} \right]^{-1} \quad (8)$$

However, the phenomenological G-matrix with the parameters determined from the effective range expansion fit leads to the somewhat lower the potential energy than the usually accepted value ~ -16 MeV.

To get a better fit we choose

$$\lambda = 2.4 \quad \beta = 1.1 \text{ fm} \quad (9)$$

These values are fairly close to the vacuum ones and provide the potential energy per particle in a good agreement with the empirical value. The parameters λ and β being substituted in the G-matrix lead to $a_m \simeq r_m \simeq 0(1)$, where a_m and r_m are the in-medium analogs of scattering length and effective radius. One notes that effective radius is much less affected by the medium effects. It is quite natural since the value of the effective radius is only weakly sensitive to the bound state at threshold and is of the “almost natural” size already in the vacuum case. The absolute value of the in-medium scattering length is considerably reduced compared to the vacuum one. It clearly indicates that, as expected, the shallow virtual nucleon-nucleon bound state is no longer present in nuclear medium. Thus, one can avoid significant part of the difficulties typical for the chiral NN problem in vacuum. Having determined the phenomenological G-matrix one can now solve the BG equation using leading and sub-leading orders of the NN effective chiral Lagrangian. The solution is similar to the vacuum case [12] and can be represented as follows

$$\frac{1}{G(k, k)} = \frac{(C_2 I_3(k, p_F) - 1)^2}{C_0 + C_2^2 I_5(k, p_F) + k^2 C_2 (2 - C_2 I_3(k, p_F))} - I(k, p_F), \quad (10)$$

where we defined

$$I_n \equiv -\frac{M}{(2\pi)^2} \int dq q^{n-1} \theta(q - p_F). \quad (11)$$

and

$$I(k) \equiv \frac{M}{2\pi^2} \int dq \frac{q^2 \theta(q - p_F)}{k^2 - q^2}. \quad (12)$$

These integrals are divergent so the renormalization should be carried out. The procedure used is similar to that adopted in Ref. [13] to study the EFT approach to the NN interaction in vacuum. We subtract the divergent integrals at some kinematical point $p^2 = -\mu^2$. After subtraction the renormalized G-matrix takes the form

$$\frac{1}{G^r(k, k)} = \frac{1}{C_0^r(\mu) + 2k^2 C_2^r(\mu)} + \frac{M}{4\pi} [p \log \frac{p_F - p}{p_F + p} - i\mu \log \frac{p_F - i\mu}{p_F + i\mu}], \quad (13)$$

One notes that in the $p_F \rightarrow 0$ limit the vacuum chiral NN amplitude is recovered. We choose the value $\mu = 0$ as a subtraction point. The μ dependence of LEC's is governed by the renormalization group equation. Now one can determine the LEC's by equating the EFT and phenomenological G-matrices at some kinematical points.

We used the values $p = \frac{p_F}{2}; \frac{p_F}{3}$ as such points. The assumed value of the Fermi-momentum is $p_F = 1.37$ fm. In the following we will omit the label “r” implying that we always deal with renormalized quantities. We found $C_0 = -1.86 fm^2$ in LO. In NLO one gets $C_0 = 2.64 fm^2$ and $C_2 = 0.84 fm^4$ so that the inclusion of the NLO corrections give rise to the approximately 40% change in the value of C_0 . It indicates that the chiral expansion is systematic in a sense that adding of the NLO terms in the effective Lagrangian results in a “NLO change” of the coefficients which have already been determined at LO. The natural size of the in-medium scattering length and moderate changes experienced by the coupling constant C_0 might, in principle, indicate the possibility of the perturbative calculations. However, in spite of this, it is still more useful to treat this problem in the nonperturbative manner. There are few reasons for the nonperturbative treatment. Firstly, the corrections themselves are quite significant. Secondly, the overall (although distant) goal of the EFT description is to derive both nuclear matter and the vacuum NN amplitude from the same Lagrangian, However, it is hard to say at what densities the dynamics becomes intrinsically nonperturbative, so it is better to treat the problem nonperturbatively from the beginning. The nonperturbative treatment may also turn out important to get the correct saturation curve since at some density lower than the normal nuclear one the scattering length starts departing from its natural value and some sort of the nonperturbative approach becomes inevitable. Thirdly, in the processes involving both the nonzero density and temperature, such as heavy ion collisions, the value of the Fermi-momentum can effectively be lowered again making the nonperturbative treatment preferable.

Let’s now calculate the potential energy per particle using the expression for the in-medium chiral NN scattering amplitude. The potential energy of nuclear matter can be evaluated from

$$U_{tot} = \frac{1}{2} \sum_{\mu, \nu} < \mu \nu | G(\epsilon_\mu + \epsilon_\nu) | \mu \nu - \nu \mu > \quad (14)$$

The summation goes over the states with momenta below p_F . Here it is seen that G amplitude plays the role of an effective chiral two-body interaction in nuclear medium. The calculations using the lowest order G -matrix result in the value $\frac{U(^1S_0)}{A} \simeq -17 MeV$. The inclusion of the next-to-leading order corrections gives rise to the value $\frac{U(^1S_0)}{A} \simeq -13.1 MeV$. One notes that both $\frac{U(^1S_0)}{A}$ and C_0 experience corrections of the same order when NLO terms are included in the effective Lagrangian. The similar calculations done in the triplet s-wave channel give rise to the value $\frac{U(^3S_1)}{A} \simeq -17.3(-13.2)$ MeV in LO (NLO). The values of the potential energy obtained with chiral approach looks quite reasonable although they are somewhat smaller than the standard values usually obtained in the calculations with the phenomenological two-body forces [14]. One can therefore conclude that there is still a room for both pionic effects and three particle correlations which should be included in a chirally invariant manner.

The validity of the EFT description is restricted by some cutoff parameter reflecting the short range physics effects. Its value deserves some comments in the context of applying of the EFT methods to nuclear matter. The scale where the EFT treatment ceases to be valid should approximately correspond to the scale of the short range correlations (SRC), that is, $\sim 2.5 fm^{-1}$. The description of SRC is hardly possible in the framework of EFT so the value $\Lambda \sim 2 fm^{-1}$ might put natural constraint on the EFT description of nuclear matter. To make the chiral expansion meaningful the chiral counting rules in nuclear matter must be established. This is still open problem. However, the above obtained results suggest that the relevant expansion parameter could be something like $\frac{\langle p \rangle}{\Lambda} \sim \frac{\langle m_\pi \rangle}{\Lambda} \sim 0.3 - 0.4$. Of course, until pion effects are taken into account this estimate can only be suggestive. Moreover, many other things remain to be done to make the qualitative description of nuclear matter possible. Beside pionic effects one needs to include many body forces and formulate chiral counting rules. One should also find a way to remove off-shell ambiguities order by order and calculate the nucleon self-energy up to a given chiral order to make EFT description of nuclear matter fully consistent.

ACKNOWLEDGMENTS

Author is very grateful for the support and warm hospitality from SRCSSM at the University of Adelaide where the initial part of this work was done.

-
- [1] A. Manohar, *Effective Field Theories*, **hep-ph/9506222**; D. B. Kaplan, *Effective Field Theories*, **nucl-th/9506035**.
 - [2] S. Weinberg, *Physica A* **96**, 32 (1979).
 - [3] J Gasser and H. Leutwyler, *Annals of Physics* **158**, 142 (1984).
 - [4] G. Ecker and M. Mojzis, *Phys. Lett. B.* **365** 312 (1996); Bernard, N. Kaiser, and U.-G. Meißner, *Nucl. Phys.* **A615**, 483 (1997).
 - [5] S. Weinberg, *Nucl. Phys.* **B363**, 3 (1991).
 - [6] D. B. Kaplan, M. Savage, and M. B. Wise, *Nucl. Phys.*, **B478**, 629 (1996); D. B. Kaplan, M. Savage, and M. B. Wise, *Phys. Lett.*, **B424**, 390 (1998), D. B. Kaplan, M. Savage, and M. B. Wise, *Nucl. Phys.*, **B534**, 329 (1998) U. van Kolck, **nucl-th/9808007**; M. C. Birse, **nucl-th/9806038**; J. V. Steele and R. J. Furnstahl, *Nucl. Phys.*, **A637**, 46(1998); T.-S. Park, K.Kubodera, D.-P. Min and M. Rho, *Phys. Rev. C* **58**, R637 (1998); G. P. Lepage, **nucl-th/9706029**; T. Mehen and I. W. Stewart, **nucl-th/9806038**.
 - [7] R. Furnstahl, B. Serot and H.-B. Tang, *Nucl. Phys.*, **A618**, 445 (1997).
 - [8] B. Lynn, *Nucl. Phys.*, **B402**, 281 (1993).

- [9] M. Lutz, Nucl. Phys., **A642**, 171 (1998).
- [10] J. Friar, **nucl-th/9804010**.
- [11] H. A. Bethe and J. Goldstone, Proc. Roy. Soc. (London) **A238**, 157 (1957).
- [12] S. Beane, T. D. Cohen and D. Phillips, Nucl. Phys., **A632**, 445 (1997).
- [13] J. Gegelia Phys. Lett., **B429**, 227 (1998).
- [14] M. I. Haftel and F. Tabakin, Nucl. Phys., **A158**, 1 (1970).